A NUMERICAL STUDY on the RESONANT SCATTERING PROCESS of RELATIVISTIC ELECTRONS via WHISTLER-MODE WAVES in the OUTER RADIATION BELT

Yuto Katoh, Takayuki Ono and Masahide Iizima

Department of Geophysics, Graduate School of Science, Tohoku University, Sendai, Miyagi, Japan

Resonant scattering processes of high energy electrons are studied numerically by using a newly developed simulation scheme where cold electrons are treated as a fluid and hot electrons are treated as particles including fully relativistic effects. The present simulation scheme enables us to investigate the resonant energy diffusion process of electrons in the inner magnetosphere, which has been difficult to study by using the PIC code. To verify the accuracy of the present scheme, elementary step of the resonant interaction of high energy electrons with whistler-mode waves are studied. The results of the experiment are consistent with the theoretical predictions and suggest that this simulation scheme has a capability to apply various problem in space physics concerning not only the resonant scattering process of high energetic electrons but the excitation of electromagnetic plasma waves and propagation process in the inhomogeneous media.

1. INTRODUCTION

As has been studied by many workers [e.g. Baker et al., 1994; Meredith et al., 2002], the flux of relativistic electrons in the outer radiation belt once decreases during the main phase of geomagnetic storms and increases again in the recovery phase. Several candidates of the mechanism have been proposed to explain the rebuilding process of the outer radiation belt; especially, the stochastic acceleration process of relativistic electrons by the resonant interaction with the enhanced plasma waves during geomagnetic storms has been studied in detail in the recent works [Summers and Ma, 2000; Miyoshi et al., 2003]. These works have their basis on the quasi-linear theory and well explain the temporal and spatial evolution of the flux enhancement. However, there still remains many fundamental physical problems concerning the acceleration processes and a numerical simulation is thought to be an important tool to obtain such physical implications. For the investigation of the elementary process of the resonant interaction, a full particle simulation must be an useful one. However it is difficult to realize the requirement of spatial scale limit for the full particle code to simulate the cyclotron resonant interaction between hot electrons and electromagnetic plasma waves. The difficulty comes
from the significant difference of their spatial scale sizes between the wave length of plasma waves and Larmor radius of thermal electrons. Let us consider that the cyclotron resonant interaction between relativistic electrons and electromagnetic plasma waves in the earth’s outer radiation belt. The wave length of whistler-mode waves is of the order of $10^3$ to $10^4$ m, while Larmor radius or Debye length of cold electrons, the medium of propagation of plasma waves, is of the order of $10^0$ m. In the study of cyclotron resonant interaction of hot electrons with the electromagnetic plasma waves, both propagation of plasma waves and the motion of cold electrons need to be solved at the same time. Because the grid spacing size must be the order of Debye length for the full particle code (PIC code), namely that $\Delta x \leq 3\lambda_e$ to avoid a nonphysical instability caused by grid spacing [Birdsall and Langdon, 1985; Omura and Matsumoto, 1993], it is not realistic to treat such a large spatial region to study the interaction with whistler-mode electromagnetic waves.

To overcome this difficulty, we have developed a new simulation scheme by utilizing a model on the basis of the concept of the hybrid algorithm. In this scheme, plasmas are separated into two components, namely low density, high energy electron component and high density, cold electron component. That is, high energy electrons are treated as relativistic particles while cold electrons are treated as a fluid, considering the difference in the spatial scales of their Larmor radii. By employing this simulation model, we have no constraint on the grid size by the small spatial scale of the gyro-motion of cold electrons. This treatment enables us to realize a simulation box with a sufficiently large spatial scale size to investigate the resonant interaction of energetic electrons with plasma waves propagating in the medium of cold electrons. As an example of the initial results, we evaluate the effect of the spectrum of whistler-mode waves by comparing the time variation of the number of the resonant particles. In this paper, we will show initial results of our simulation model and discuss the capability of the model for treating a basic physical process of resonant interaction. The results show that we can treat the acceleration process under the realistic magnetospheric condition by using our simulation model. Hereinafter, we refer to this hybrid model as "Electron Hybrid Model".

2. DETAILS OF THE MODEL

In our newly developed simulation scheme, the high energy electrons are treated as particles and the cold electrons as a fluid. Numerical simulation studies on
the basis of similar approaches have been made by Rathmann et al. [1978] and Omura and Matsumoto [1982], named Long-Time-Scale Code; and Taguchi et al. [2001], named Hybrid Darwin Code. Compared with these already established simulation model, our simulation model has several advantages: First, particle motions of high energy electrons are treated including fully relativistic effect. The evolution of the electromagnetic field in time and space are directly solved without any assumptions by using Maxwell’s equations. Thus, it can be said that our simulation model describes realistic energy transfer between high energy electrons and plasma waves propagating in the medium of cold electrons. This is one of the most significant advantages given by the present model; this model enables us to treat the wide wavenumber range of plasma waves covering excitation and propagation of free space electromagnetic waves and their cutoff process in inhomogeneous media.

Basic equations in this model to describe the particle motions and electromagnetic fields are as follows:

\[
\begin{align*}
\frac{\partial \mathbf{v}_f}{\partial t} &= - (\mathbf{v}_f \cdot \nabla) \mathbf{v}_f + \frac{q}{m_e} (\mathbf{E} + \mathbf{v}_f \times \mathbf{B}), \\
\frac{\partial \rho_f}{\partial t} &= - \nabla \cdot (\rho_f \mathbf{v}_f), \\
\frac{\partial \mathbf{B}}{\partial t} &= - \nabla \times \mathbf{E}, \\
\frac{\partial \mathbf{E}}{\partial t} &= \frac{1}{\mu_0 \varepsilon_0} \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0} \mathbf{J}, \\
\mathbf{J} &= \mathbf{J}_f + \mathbf{J}_p, \\
\frac{d \mathbf{p}_p}{dt} &= q (\mathbf{E} + \mathbf{v}_p \times \mathbf{B}), \\
\frac{d \mathbf{E}_{\text{kin}}}{dt} &= \mathbf{v}_p \cdot \frac{d \mathbf{p}_p}{dt},
\end{align*}
\]

and

\[
\mathbf{p}_p = \frac{\mathbf{v}_p c^2}{c^2} \mathbf{E}_{\text{kin}}
\]

where subscripts \( f \) and \( p \) denote the quantities associated with the cold (fluid) component and hot (particle) component of electrons, respectively; \( \mathbf{E}_{\text{kin}} \) is kinetic energy of the hot electron including its rest mass energy; and \( q \) and \( m_e \) represent charge and mass of electron, respectively. In the present study, we assume that ions are stationary background in the simulation system with a density satisfying quasi-charge-neutrality. Since the time scale in the present study is the order of the electron gyro-period, it is adequate to employ this assumption. The current densities of cold electron \( \mathbf{J}_f \) and hot electron \( \mathbf{J}_p \) are calculated separately from the motions of each electron species. A two-step
Lax-Wendroff scheme is utilized to calculate the time development of the electromagnetic field and fluid motion of cold electrons. PIC scheme is applied for the hot electron particles and their motions are calculated by using a leap-frog method. To implement the simulation code, each physical value is normalized to be a dimensionless quantity; time is normalized by electron gyro-period $T_{ce}$, velocity and length are normalized by speed of light $c$ and $cT_{ce}$, respectively.

Simulation system is separated into three regions as shown in Figure 1; namely, the physical region, the wave source region, and the damping region. High energy electrons are distributed in the physical region by employing a periodic boundary condition. Since the location of the boundary of hot electrons is different from that of cold electrons, to avoid the reflection of waves, a masking method is applied to the electric field caused by the current density of hot electrons near the boundary of the physical region. Plasma waves are assumed to propagate into the physical region from the source region (external source) along the direction of X-axis. In the wave source region, electromagnetic plasma waves are generated by an artificial oscillation of the electric field.

Figure 1

Numerical experiments have mainly been carried out to simulate the basic process of the resonant interaction between high energy electrons and whistler-mode plasma waves. In these simulation runs, we choose the system length of physical region $L_x$ as 4096 $\Delta x$ aligned to the initial magnetic field direction while grid spacing $\Delta x$ as $1.0 \times 10^{-2} cT_{ce}$ and time step $\Delta t$ as $7.5 \times 10^{-3} T_{ce}^{-1}$, which satisfy the time and spatial limits necessary for the precise simulation of plasma wave. The validity in the choice of these values are examined through the verification of exact propagation of plasma waves. In the present simulation the plasma parameter $f_p/f_e$ is set to be $f_p/f_e = 1.0$, corresponding to the parameter at the outside region of the plasmapause; these grid spacing and time step width are corresponding to 100 m and 0.268 $\mu$sec, respectively, for the real spatial length and time interval.

It is noted that the number of grid points required for a full particle code simulation with the same system length is ten times as large as that of our simulation scheme, since Debye length of cold electrons is 3 m in this case. This advantage of Electron Hybrid Model will be more clearly recognized in the case of 2D or 3D simulation.
3. RESULTS OF NUMERICAL EXPERIMENT

To verify the validity of the present Electron Hybrid Model, test simulations are performed concerning the resonant interaction process of the high energy electrons with whistler-mode waves. To see the basic processes of the resonant interaction at first, the distribution function of high energy electrons is assumed to be a shifted bi-maxwellian type centered at a half of the speed of light corresponding to the energy of 79 keV. They are uniformly distributed in the physical region with their initial pitch angles and gyrophases forming an isotropic distribution (Figure 2). The plasma parameter of the background cold plasma is assumed to be as $f_p/f_c = 1.0$, which represents the typical plasma parameter of the outside region of plasmapause. The number density of high energy electrons is set to be $10^{-6}$ of the background cold electrons; 1,310,720 superparticles have been used for the calculation of the motion of high energy electrons.

As a first step, we compare the scattering processes by the monochromatic whistler-mode wave and by the whistler-mode wave with a banded spectrum characters. The monochromatic wave has a frequency of 0.5 $\Omega_e$, while the whistler-mode wave with a banded spectrum consists of waves continuously distributed from 0.3 to 0.6 $\Omega_e$. In each case, effective values of the wave amplitudes were set to be 1 mV$_{\text{rms}}$/m. We conducted each experiment up to a time of $7.5 \times 10^3$ gyro-cycle of electrons which corresponds to 270 msec time interval in the earth's inner magnetosphere.

The distributions of resonant particles in the velocity space at $7.5 \times 10^3 T_e$ are shown in Figure 3(a) and (b), where these particles are picked up according to the criterion that the speed and pitch angle change from their initial conditions up to a value of 0.1% and 0.1 degree, respectively. In both cases, as shown in Figure 3(a) and (b), resonant particles are distributed on or within the cyclotron resonant curves. We have estimated the resonant curves from $\omega - k_\parallel v_\parallel = \Omega_e/\gamma$, where $v_\parallel$ is field aligned component of the velocity of resonant particles and $\gamma$ is the Lorentz factor.

Figure 2 Figure 3

Figure 4 represents the variations in the speed of the high energy electrons from their initial speed at 1500, 3750 and 7500 $T_e$. In early stage of the experiment, velocities of resonant particles in the case of the monochromatic wave show a more rapid change than the banded waves, until reaching a quasi-stationary state at $1500 \, \Omega_e^{-1}$ (Fig 4(a)). Although the velocity of resonant particles changes moderately in the case of waves with finite band width, the maximum variance in
the velocity \(|\delta v| \sim 8.0 \times 10^{-3}c\) is rather larger than
the monochromatic wave case \((-\delta v| \sim 6.0 \times 10^{-3}c\)
as shown in Fig 4(c). This result can be explained by the continuous resonant scattering effect due to the
broadness of the wave spectrum of whistler-mode wave.
The resonant interaction caused by the wave with a fre-
quency of \(f_a\) affects resonant particles which satisfy the
resonance condition with \(f_a\) wave; the resonant par-
ticles are then accelerated (decelerated) and deviates
from the resonance condition with \(f_a\) wave. In the case
of the banded whistler-mode wave, these deviated par-
ticles subsequently interact with another wave with a
frequency of \(f_b = f_a + \delta f\); the effective acceleration
(deceleration) of resonant particles occurs in the case of
a wave with a finite band width.

Figure 4

The efficient resonant scattering process for the case
of the wave with banded wave spectrum is also indi-
cated in the energy transfer process between whistler-
mode waves and energetic particles. In Figure 5, we
show the time variations of the kinetic energy density of
the energetic electron species for both types of waves.
The kinetic energy density of energetic electrons con-
tinuously increases by the effect of the successive reso-
nant interaction for the case of banded wave, whereas
the kinetic energy slightly decreases for the case of the
monochromatic wave. These simulation results suggest
a possibility that waves with a finite band width such
as turbulent waves induce efficient scattering of reso-
nant particles in the phase space; that is, plasmaspheric
hiss is an efficient energy source for the electrons, while
man-made, monochromatic transmitter signals actually
remove energy from plasma.

Figure 5

4. DISCUSSIONS

On the basis of the present simulation results, it has
been demonstrated that our simulation scheme has a
enough capability to understand the detailed features
of the wave-particle interaction processes in the space
plasma. In Fig 3(a), there are ‘resonant’ particles which
are not lying on the resonant curve estimated from the
frequency of the injected whistler-mode wave exactly.
The existence of ‘resonant’ electrons can be explained
by the effect of the generated plasma waves by the
high energy electrons in the physical region. In the
present simulation, the velocity distribution function of
the high energy electrons is assumed to be the shifted-
maxwellian type distribution function, and such elec-
trons act as high energy beams in the simulation space.
The beam distribution of electrons possibly excite the
plasma waves due to the beam-type instability. To estimate the amplitude of such self-excited plasma waves, we performed another simulation without injecting the whistler-mode waves; the initial parameters of electrons and the condition of the simulation system are set to be the same as those represented in the previous section. Figure 6 shows the $\omega - k$ diagram of the plasma waves generated in the physical region. In the simulation result, whistler-mode waves on the branch of the parallel propagating mode are excited with the wave amplitude up to several $\mu V/m$, while their wave frequencies satisfy the first order cyclotron resonance condition with the energetic electrons, i.e., $\omega - k|v| - \Omega_e/\gamma = 0$. This result shows that the whistler-mode waves are also excited through the wave-particle interaction, the excited whistler-mode waves then interact with the another resonant electrons during the propagation in the simulation space. The origin of the decrease of the total kinetic energy in the case of monochromatic wave shown in Figure 5 can also be explained when considering the effect of the self-excitation of whistler-mode waves. The kinetic energy of energetic particles is transferred to the wave energy of excited whistler-mode waves, and the energy escapes from the simulation system because the open boundary condition is applied in the present study. This result indicates that the present simulation scheme precisely treat the total wave-particle interaction process and has an enough capability for an application to various problems in space physics concerned with wave-particle interaction including propagation and excitation of electromagnetic plasma waves.

In the previous section, we have compared the scattering processes by the monochromatic whistler-mode wave and by the whistler-mode wave with a banded spectrum characters. The simulation results suggest that the wave spectrum of whistler-mode waves play an essential role in the resonant scattering process. In the case of whistler-mode waves with finite band width, the maximum variance in the speed of resonant particle, $|\delta v| \sim 8.0 \times 10^{-3}c$ at 7500 $T_e$, which corresponds to the change in kinetic energy of 2.8 keV for the electron of initial energy of 79 keV; this is equivalent to the temporal energizing rate of 10.4 keV/sec. Although this is an extremely high acceleration rate, the effects of both the given finite band width of the wave spectrum and the homogeneity in the wave propagation media assumed in the present simulation system restrict the energizing of resonant electrons up to 170 keV. This estimation is derived from a diffusion curve in the velocity space in the present study, following the work of Summers et
During the resonant scattering process in the homogeneous system, resonant particles move along the diffusion curve according to the change in their resonance condition. In the present simulation, since we assumed that the width of the wave spectrum is limited from $0.3\Omega_e$ to $0.6\Omega_e$, the motion of resonant particles in the velocity space is restricted on the finite element of the diffusion curve. Therefore, in the homogeneous system, energization of resonant particles saturates at an appropriate value determined by the width of the wave spectrum. This limitation of energization will be violated by taking into account for the inhomogeneity of the background magnetic field or the effect of the pitch angle scattering caused by other plasma waves. Since these effect cause the modulation of the motion of resonant particles in the velocity space, the motion of resonant particles, or diffusion process then shows a stochastic nature.

In this paper, we have presented the results of our newly developed simulation scheme and concluded that the present model accurately simulates the resonant scattering process of high energy electrons. The resonant scattering process via whistler-mode wave is recognized as a strong candidate in the rebuilding process of the outer radiation belt during the geomagnetic recovery phase. Previous theoretical analyses and observational evidences revealed the importance of the effect of resonant interaction. In this current of study, the quantitative estimation of acceleration process and the verification of diffusion coefficients derived from quasi-linear theory have been recognized as an important problem. The diffusion coefficients of high energy particles during resonant interaction with plasma waves has been discussed by various authors on the basis of the quasi-linear theory [e.g., Lyons, 1974; Albert, 1999]. To understand the details of the properties of pitch angle and energy diffusion processes of energetic particles through the wave particle interactions with the plasma waves, the diffusion coefficient in the phase space is especially important and should be accurately evaluated considering the wave particle interaction processes by using realistic plasma wave parameters. The previous studies explained the response of high energy particles through the resonant scattering with enhanced whistler and ion cyclotron waves by adopting the diffusion coefficient estimated for a high density plasma condition $((\omega_{pe}/\Omega_e)^2 \gg \omega/\Omega_e)$ for the region of inner plasmasphere [Lyons, 1974]. For the stochastic acceleration processes with whistler-mode waves, however, a condition of low plasma density, which is typical for the case of the outside region of plasmapause, is required.
for effective acceleration by the whistler-mode waves; therefore it is important to estimate the accurate diffusion coefficients on such realistic plasma parameters as proposed by Horne et al. (2003). The results of in-situ observation in the inner magnetosphere also reveal the real distribution function and pitch angle distribution of high energy particles which is not the shifted bi-Maxwellian type used in this initial study. To discuss the realistic acceleration process of high energetic electrons, it is necessary to discuss the resonant scattering process by considering the realistic distribution of energetic electrons as well as the realistic plasma parameters as observed in the magnetosphere. By employing the present simulation model, it will be possible to investigate the diffusion process of resonant electrons and estimate their diffusion rate under such a real magnetospheric plasma condition; these works are important for future studies.

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M. Iizima and T. Ono, Department of Geophysics, Graduate School of Science, Tohoku University, Sendai, Miyagi 980-8578, Japan. (e-mail: iizima@stpp1.geophys.tohoku.ac.jp; ono@stpp3.geophys.tohoku.ac.jp)

Y. Katoh, Research Institute for Sustainable Humanosphere, Kyoto University, Uji, Kyoto 611-0011, Japan. (e-mail: yuto@rish.kyoto-u.ac.jp)

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1Now at Research Institute for Sustainable Humanosphere, Kyoto University, Uji, Kyoto, Japan.
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(a) Whistler: $\omega = 0.5 \Omega_c$, Resonance Particle

(b) Whistler: $\omega = 0.3-0.6 \Omega_c$, Resonance Particle